Factor Quadratic Expressions of the Form $x^2 + bx + c$

Lesson objectives

- I know that quadratic expressions represent areas
- I know that the factors of a quadratic represent the length and width of a rectangle
- I know how to factor a quadratic using a chart
- I know how to factor a quadratic using decomposition

1.1

Lesson objectives | Teachers' notes | Lesson notes

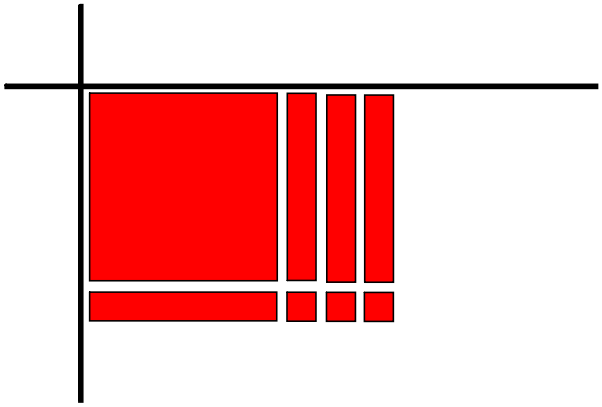
MHR Page 240 #s 2, 3ade, 4bcf, 5, 7cdef, 8ab, 9cd, 11, 13 & 14

So how do we go from standard form to factored form?

If we think of this in terms of the tiles...

We have the area, so we have to find the length and width.

Remember that the area is a rectangle. Eg: $x^2 + 4x + 3$



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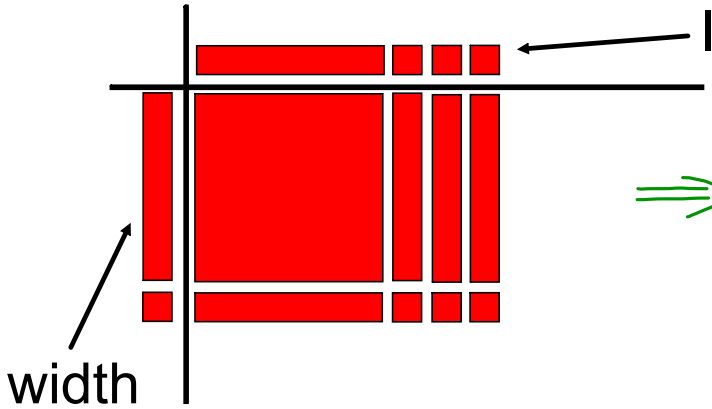
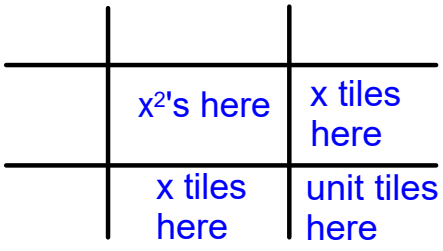


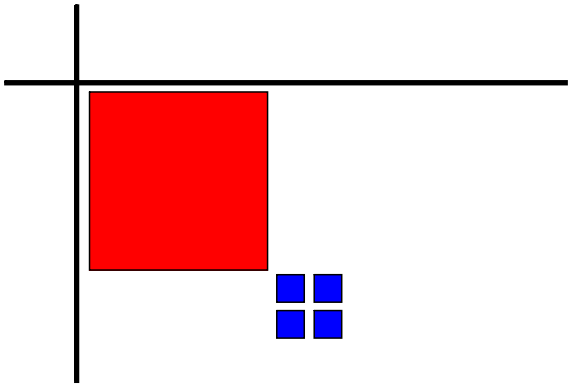
$\Rightarrow (x+3)(x+1)$

Setting up the "Rectangle"

When our unit tiles are negative one set of x tiles will be negative and the other will be positive. We will have to add "zero" pairs to the original question to find the "rectangle".



Eg: $x^2 - 4$



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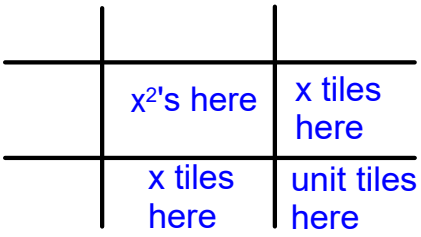

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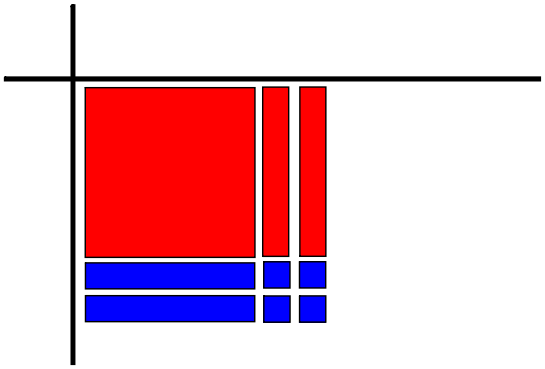

Zero pairs are terms that equal and opposite.They cancel each other out, but can be used to fill the space to complete the rectangle.

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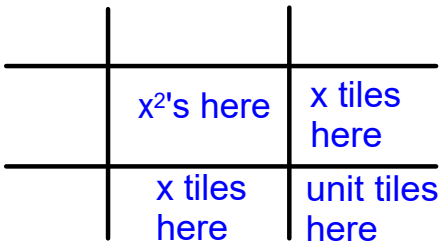

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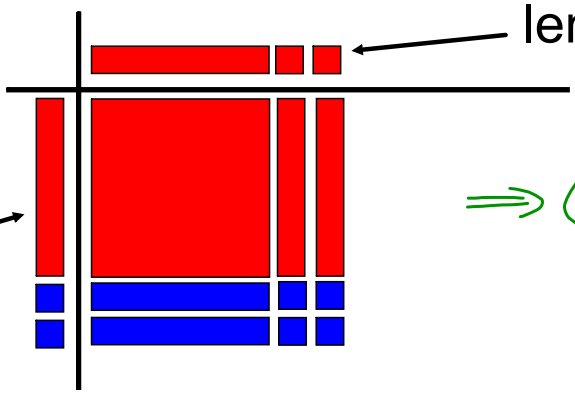


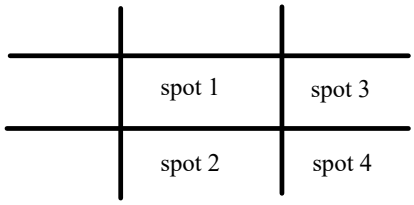
$\Rightarrow (x+2)(x-2)$

Factoring Using a Chart

1. Check your trinomial to see if you can common factor first.
2. Write the x^2 term in position #1. Write the constant term in position #4
3. Multiply the coefficient of x^2 and the constant term.

You need to find two numbers that multiply to make (a)(c) and add to make b (the coefficient of x).

4. The two numbers you get in step 3 become your coefficients of x for position #2 and #3 in your chart.
5. Common factor horizontally twice and common factor vertically twice. You will now have your factors.



$x^2 + 7x + 12$

Find two numbers that multiply to make 12 and add to make 7

	x	3
x	x^2	$3x$
4	$4x$	12

$\Rightarrow (x+4)(x+3)$

Factors

1×12

2×6

3×4

When multiplying to make a positive, both numbers are positive **OR** both are negative. Check the b value to see which it will be.

$x^2 + 3x - 18$

Find two numbers that multiply to make -18
and add to make 3

Factors

-1×18
-2×9
-3×6

	x	-3
x	x^2	$-3x$
6	$6x$	-18

$\Rightarrow (x+6)(x-3)$

When multiplying to make a negative, one number is positive and the other is negative. If the b value is positive the larger number will be positive. If the b value is negative the larger number will be negative.

The Algebra of Decomposition

Decomposition is the purely algebraic method of factoring trinomials. It starts with the same steps as the chart method:

1: Determine two numbers that add to "b" and multiply to the product "ac".

2: Decompose the middle term. In other words, use the two numbers you found in step one as the coefficients of x and rewrite x as two terms.

3: Group the first two terms and the last two terms.

4: Find the common factor of the first two terms. Find the common factor of the second two terms. Hint: (what is left after the common factor is removed from each group should be the same)

5: Common factor the two terms you now have.

6: You now have factored form.

$x^2 + 4x + 6$

Factors

1×6

2×3

?

As there is no pair that multiplies to make 6, that also adds to make 4

$\Longrightarrow$ Not factorable

$x^2 - 29x + 28$

Factors

-1×-28

-2×-14

-4×-7

$= x^2 - x - 28x + 28$

$= x(x-1) - 28(x-1)$

$= (x-1)(x-28)$

$x^2 - 4x - 21$

$= x^2 + 3x - 7x - 21$

$= x(x+3) - 7(x+3)$

$= (x+3)(x-7)$

Factors

1×-21

3×-7